Point Group Tables 1/2

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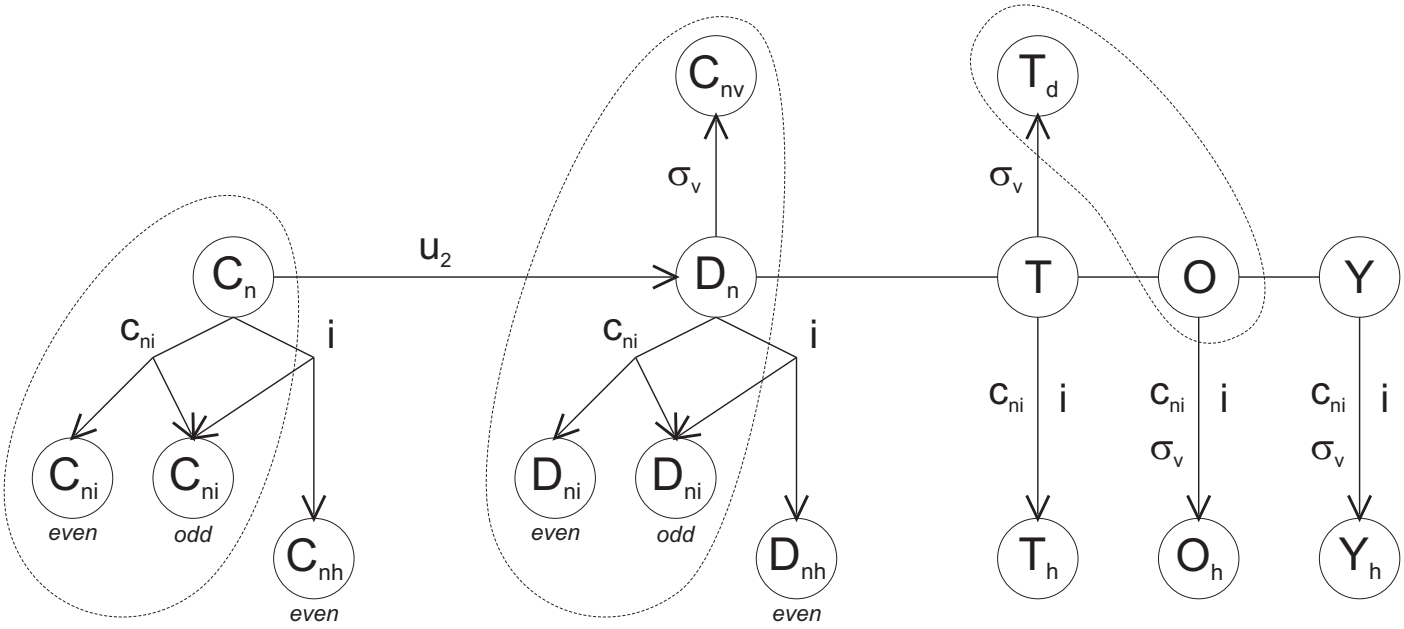

Figure 1: The hierarchy of point groups (isomorphic groups are combined).

Table 1: Correspondence between Schoenflies and short Hermann–Mauguin notations (duplicates are in parentheses).

Scho	C_n	C_{ni}	C_{nh}	D_n	D_{nd}	D_{nh}	C_{nv}	E_n	E_{nh}	E_{nv}	
H-M odd	n	-n		n2	-nm	-(2n)m2	nm				
H-M even	n	-n	n/m	n22	-(2n)2m	n/mmm	$n$mm				
1	1	-1	m	(2)	(2/m)	(2/m)	(m)				
2	2	(m)	2/m	222	-42m	mmm	mm2				
3	3	-3	-6	32	-3m	-6m2	3m	23	m-3	-43m	T
4	4	-4	4/m	422	-82m	4/mmm	4mm	432	m-3m		O
5	5	-5	-10	52	-5m	-10m2	5m	25	m-5		Y
6	6	(-6)	6/m	622	-122m	6/mmm	6mm				
...	...	...	...	...	...	...	...				
∞	∞	∞/m	∞/m	∞2	∞/mm	∞/mm	∞mm	$\infty\infty$	$\infty\infty$m	$\infty\infty$m	

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Table 2: Representation of point groups. Here $q = \mathrm{e}^{2\pi \mathrm{i}/3}$. Generators are in bold. For direct products with the inversion group, only the list of additional classes is given, the representations themselves can be obtained using examples of C_i, C_{3i}, C_{2h}, D_{3i}. The groups C_2 and C_{2h} are duplicated for convenience. Upper indexes denote alternative group names, in this case the inversion axis should be replaced by the corresponding reflection axis as follows: $c_{3i} = s_6^{-1}$, $c_{4i} = s_4^{-1}$, $c_{6i} = s_3^{-1}$. Self-representations: $C_{3,4,6}$ and $D_{3,4,6}$ – E, $T_{,h}$ – F, T_d – F_2, $O_{,h}$ and $Y_{,h}$ – F_1.

C_1		e	
	C_i	e	$\mathbf{i}$
	C_s	e	$\sigma_\mathbf{h}$
	C_2	e	$\mathbf{c_2}$
A	A_g A' A	1	1
	A_u A'' B	1	-1
	C_{2h}	$\mathbf{i}$	σ_h

C_{4i}^{2d}	e	c_2	$\mathbf{c_{4i}}$	c_{4i}^{-1}
C_4	e	c_2	$\mathbf{c_4}$	c_4^{-1}
A	1	1	1	1
B	1	1	-1	-1
E	1	-1	i	$-$i
	1	-1	$-$i	i
C_{4h}	$\mathbf{i}$	σ_h	c_{4i}	c_{4i}^{-1}

C_3		e	$\mathbf{c_3}$	c_3^{-1}			
	C_{3i}^{3d}	e	c_3	c_3^{-1}	i	$\mathbf{c_{3i}}$	c_{3i}^{-1}
	C_{6i}^{3h}	e	c_3	c_3^{-1}	σ_h	$\mathbf{c_{6i}}$	c_{6i}^{-1}
	C_6	e	c_3^{-1}	c_3	c_2	$\mathbf{c_6}$	c_6^{-1}
A	A_g A' A	1	1	1	1	1	1
E	E_g E' E_2	1	q	q^2	1	q	q^2
		1	q^2	q	1	q^2	q
	A_u A'' B	1	1	1	-1	-1	-1
	E_u E'' E_1	1	q	q^2	-1	$-q$	$-q^2$
		1	q^2	q	-1	$-q^2$	$-q$
	C_{6h}	$\mathbf{i}$	c_{3i}^{-1}	c_{3i}	σ_h	c_{6i}	c_{6i}^{-1}

C_2			e	$\mathbf{c_2}$	
	C_{2h}		e	$\mathbf{c_2}$	$\mathbf{i}$ σ_h
	C_{2v}		e	$\mathbf{c_2}$	$\sigma_\mathbf{v}$ σ_v'
	D_2		e	$\mathbf{c_2}$	$\mathbf{u_2}$ u_2'
A	A_g	A_1 A	1	1	1 1
B	B_g	B_1 B_2	1	-1	1 -1
	A_u	A_2 B_3	1	1	-1 -1
	B_u	B_2 B_1	1	-1	-1 1
	D_{2h}		$\mathbf{i}$	σ_h	σ_v σ_v'

C_{4v}	e	c_2	$2\mathbf{c_4}$	$2\sigma_\mathbf{v}$	$2\sigma_v'$
D_{4i}^{2d}	e	c_2	$2\mathbf{c_{4i}}$	$2\mathbf{u_2}$	$2\sigma_v'$
D_4	e	c_2	$2\mathbf{c_4}$	$2\mathbf{u_2}$	$2u_2'$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
B_1	1	1	-1	1	-1
B_2	1	1	-1	-1	1
E	2	-2	0	0	0
D_{4h}	$\mathbf{i}$	σ_h	$2c_{4i}$	$2\sigma_v$	$2\sigma_v'$

C_{3v}			e	$2\mathbf{c_3}$	$3\sigma_\mathbf{v}$			
D_3			e	$2\mathbf{c_3}$	$3\mathbf{u_2}$			
	D_{3i}^{3d}		e	$2c_3$	$3\mathbf{u_2}$	i	$2\mathbf{c_{3i}}$	$3\sigma_v$
	D_{6i}^{3h}		e	$2c_3$	$3\mathbf{u_2}$	σ_h	$2\mathbf{c_{6i}}$	$3\sigma_v'$
	C_{6v}		e	$2c_3$	$3\sigma_\mathbf{v}$	c_2	$2\mathbf{c_6}$	$3\sigma_v'$
	D_6		e	$2c_3$	$3\mathbf{u_2}$	c_2	$2\mathbf{c_6}$	$3u_2'$
A_1	A_{1g}	A_1' A_1	1	1	1	1	1	1
A_2	A_{2g}	A_2' A_2	1	1	-1	1	1	-1
E	E_g	E' E_2	2	-1	0	2	-1	0
	A_{1u}	A_1'' B_1	1	1	1	-1	-1	-1
	A_{2u}	A_2'' B_2	1	1	-1	-1	-1	1
	E_u	E'' E_1	2	-1	0	-2	1	0
	D_{6h}		$\mathbf{i}$	$2c_{3i}$	$3\sigma_v$	σ_h	$2c_{6i}$	$3\sigma_v'$

T	e	$4\mathbf{c_3}$	$4c_3^{-1}$	$3\mathbf{u_2}$
A	1	1	1	1
E	1	q	q^2	1
	1	q^2	q	1
F	3	0	0	-1
T_h	i	$4\mathbf{c_{3i}}$	$4c_{3i}^{-1}$	$3\sigma_v$

T_d	e	$8c_3$	$3u_2$	$6\mathbf{c_{4i}}$	$6\sigma_\mathbf{v}$
O	e	$8u_3$	$3c_2$	$6\mathbf{c_4}$	$6\mathbf{u_2}$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
F_1	3	0	-1	1	-1
F_2	3	0	-1	-1	1
O_h	i	$8u_{3i}$	$3\sigma_h$	$6\mathbf{c_{4i}}$	$6\sigma_v$

Y	e	$12\mathbf{c_5}$	$12c_5^2$	$15\mathbf{u_2}$	$20u_3$
A	1	1	1	1	1
F_1	3	$\frac{1+\sqrt{5}}{2}$	$\frac{1-\sqrt{5}}{2}$	-1	0
F_2	3	$\frac{1-\sqrt{5}}{2}$	$\frac{1+\sqrt{5}}{2}$	-1	0
G	4	-1	-1	0	1
H	5	0	0	1	-1
Y_h	i	$12\mathbf{c_{5i}}$	$12c_{5i}^3$	$15\sigma_v$	$20u_{3i}$